Monitor your loss provision!

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1 Introduction

The explicitness of how loss provisions are ascertained will become a compulsory activity for insurance companies in the not too distant future. In the past safe-proven loss provisions, which were ascertained in a stable way, would suffice. This will change, however, to the objective testing of the very content of prudence. Objective methods are therefore needed and they can be found in probability theory and mathematical statistics.

The classical elaboration in actuarial science has resulted in a huge amount of literature on ruin theory. The main drawback is that it is not readily applicable in daily work and for the year account. We suggest a new approach, which is not hampered by inappropriate theory. This focuses in particular on unbiased prediction, prudent provisions and the embedding of solvency, as shown in figure 1. The starting point of our method, which uses matrix algebra, econometrics and time series analysis, is formed by runoff tables, which are available at insurance companies. Below we give an outline of our novel approach and compare it with other approaches, such as logarithmic transformations and the - in our view dubious - calendar time paradigm. At the end you will find an applied example of a runoff table with loss years and settlement duration quarters.

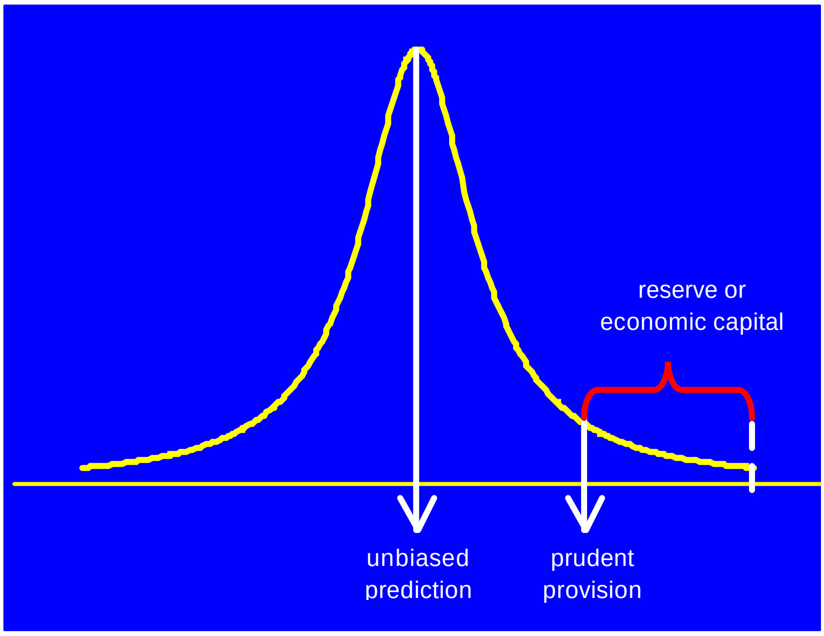

Figure 1: Aspects of uncertainty

2 Premises

Stochastic models should reflect the core aspects of the risk and settlement process. With such a model we can objectify a prudence margin: together with an unbiased prediction we may choose a quantile, for instance of order 0.8, which gives a prudent provision. In four cases out of five this prudent provision will ultimately result in a positive runoff result. A loss provision ascertained in this way appeals to the new accounting requirements.

What should we require of a good stochastic model? First of all that it has professional requirements, such as a logical and efficient specification, consistent estimators and predictive power. Furthermore, transparency of the results of the model will ease communication with management and decision makers. A model that does not will fail.

3 Aspects of the model

3.1 Interpreting marginal totals

Let us focus on a runoff table with loss years on the vertical axis and settlement duration on the horizontal axis, as in figure 2.

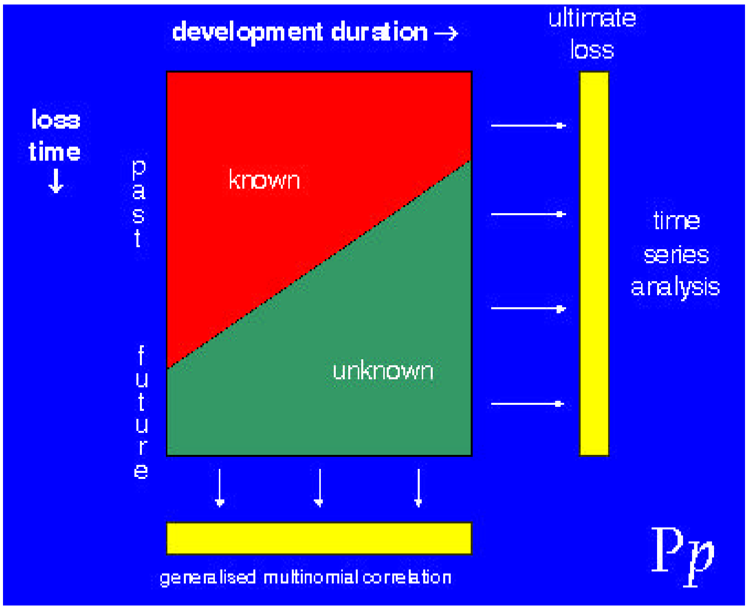

Figure 2: Visualization of the model aspects

As long as this triangle has not grown out into a square, the insurer faces a problem in ascertaining a loss provision for the outstanding loss years. Let us now place ourselves in the distant future when this triangle is a completed square. No further loss provisions are needed and we can form two marginal totals. Taking the column of row totals results in a time series of aggregate loss by loss year. This time series can be related to a time series of earned premiums by loss year, which gives a time series of loss ratios. A graph of this ultimate loss ratio gives an impression of a fluctuating pattern about a horizontal level, a trend or a sudden change. In the case of quarterly data seasonality may also be observed. The evolution of the loss ratio is the first concept that will appeal to insurance management. For a statistician it forms a convenient asset for stochastic modelling of the inner part of the square. Indeed, its row total should correspond to the outcome of a time series analysis for which there is an abundance of literature. Forming a row of column totals gives a benefit-weighted settlement pattern. This pattern results from a variety of waiting and service time processes, which starts with the reporting lag. Notwithstanding the yearly or quarterly nature of the runoff triangle, this settlement duration is a time continuous process. We may model this process by using familiar formulae as known from our statistical handbooks. From there we achieve a stabilised histogram for the settlement duration in the observed discretised time.

To put it another way: the cells of the square are random variables. Their means have a row and a column effect. The row effects result from time series analysis. The column effects follow from a smooth probability density function. This basically differs from methods where row and column effects each have their own structural parameter, such as chain ladder and loglinear models with dummy variables. For a sparse observed uppertail of the empirical settlement duration, which may contain tiny and negative observations, in particular the density function will act as a balanced vehicle, which serves for extrapolating outside the finite runoff triangle.

3.2 Aggregation level

Traditionally, runoff triangles are on a yearly basis. Quarterly and monthly observational plans have now become a practical possibility. However, it is not a law of nature that such a fine level requires the same level for statistical analysis. The choice of an optimum length of the loss and settlement duration period is also influenced by the content of the cell. By not choosing it too small we may achieve a feeling of attraction to the normal distribution for the probability distribution for the loss in a single cell. Nor is there any need for all the columns to be of equal width. This allows us to group the cells of the uppertail into a single cell with empirical mass. Where we do not know or trust the incremental evolution for old accounting years, we can group them into single cumulative observations.

3.3 Multivariate normal manipulation

The specification of the means, variances and covariances for all cells, whether observed or still random, can be derived along the lines mentioned above. The aggregation of cells, as for sparse uppertails, implies a corresponding adjustment of these moments. Mathematically, this is most easily done by linear transformations from matrix algebra. This is a powerful and flexible approach, which feels particularly at ease with a multivariate normal distribution for the various cells of the runoff square. Parameter estimation is a matter of formulating the likelihood function for observed and grouped data. This is easy for a multivariate normal density. The parameters enter as nonlinear functions for the means and (co)variances and estimation is a matter of numerical optimisation. When we have grouped our data for estimation, we can use the parameter estimates obtained for actuarial application at the original level of loss and settlement duration period again.

4 The impact of estimation uncertainty

Optimisation of the logarithm of the likelihood function gives maximum likelihood estimates. At the optimum the negative inverse of the Hessian matrix gives an estimate for the covariance matrix of these estimates, which are asymptotically multivariate normal distributed. This distribution acts as a weighting distribution for the predictive distribution for the unknown cells, conditionally on the known cells and the unknown parameters. The unknown parameters and their uncertainty are mixed away in this manner. The effect of this operation is an increase of the covariance matrix of the prediction errors and a change from a multivariate normal distribution to a multivariate Student distribution. A Student distribution has thicker tails than a normal distribution, which increases prediction intervals, especially in cases of extreme probability levels.

5 Comparison with other approaches

Our method is strongly influenced by time continuous thinking. This manifests itself not only in the density function for the settlement duration, but also in the possibility of aggregating cells into larger ones for estimation purposes and disaggregating again for actuarial purposes. For this reason there is not a special case in our model, which boils down to a chain ladder approach. It conflicts with the aggregation-disaggregation property. In order to have something that may come close to the settlement duration effects of a chain ladder, we have designed a piecewise linear density function that can have a lot of a priori specified knot points.

An important aspect of our model is that it does not need a nonlinear transformation, as for lognormal assumed cells. A logarithmic transformation to enable linear least squares parameter estimation becomes sensitive upon applying the exponential transformation to obtain actuarial predictions. Not only is unbiasedness at risk, but also results are sensitive for the correctness of the lognormal specification. This manifests itself in large levels for predictions and upper quantiles. Excessively high levels for prudent loss provisions are consequently a real risk.
The handling of negative cells forms a problem for lognormality. Our normal distribution, whose appropriateness can be increased by aggregation of cells, smoothes this problem away.

For deliberate reasons, our model does not have a diagonal calendar time effect. It is our impression that such an autonomous effect has been put forward in a discrete time environment, where inflation and shock-wise changes in the settlement process are the classical driving forces. When we consider it in a more fundamental time continuous setting, such a diagonal effect breaks down or can be avoided. It is well known that a constant inflation effect can be absorbed in the row and the column effect. Of course, over time inflation will be accompanied by some noise. In this case the very calendar time effect will be determined by this very noise. This does not seem smart or efficient. In case inflation over time does not fluctuate about a constant level, the preferred approach should be to deflate the runoff triangle by the inflation index development known from other sources. In any event, inflation as a calendar time effect can be avoided and does not need additional parameter estimation. In the event of calendar time effects driven by shocks due to changes in the settlement duration process, procedures, legislation, etc., there may be a genuine problem. There will be a change in the benefit-weighted settlement duration density. Whether this will likewise apply to the older loss periods is not clear. If it does, the effect does not work out as a universal multiplication factor along the accounting year diagonal, which is the basic calendar time model.

6 Applications

As an example we can consider a motorcar portfolio generated by independent agents where we encounter a typical runoff triangle with loss years and settlement duration in quarters. The effect of changing actuarial control parameters such as discount rate, prudence as a probability and the length of the loss period can be seen in an interactive way. This actuarial phase has been preceded by a statistical modelling phase, which can be outlined as follows.

We need parameter estimates for the time series and the settlement duration density. The analysis of residuals should look good and the model should have predictive power. For monitoring the runoff triangle we have to verify the prediction errors along the accounting year diagonal. These prediction errors should fluctuate between acceptable levels. As additional knowledge on the portfolio we know that the insurance cover conditions changed in a restrictive way at the beginning of 1994 and 2000. This induced a constant loss ratio model with two (downward) jumps in the horizontal level. Furthermore, scrutinising the loss provision for old cases revealed that after seven years they assumed reliable values. We grouped the uppertail of payments with settlement duration greater than six years together with the outstanding loss provisions to single cells for these loss years.

The settlement duration density was chosen as a power gamma density. This is a density with three parameters, which contains the lognormal, gamma, Weibull and others as a special case. Its uppertail can vary from light to heavy, like Pareto. The maximum likelihood estimates, standard deviations and correlation matrix can be viewed in the interactive report. For the loss ratio we see that it moved from a level of 74% through a level of 64% to a current long-run level of 56%.

If we compare the observations with their expectations according to the estimated model and divide them by their specific standard deviations, we obtain standardised residuals. For a good model these standardised residuals should have values of about 0. Overall, their mean, standard deviation, skewness and kurtosis should have values of about 0, 1, 0 and 3. The payments in the fourth quarter of 1994 regarding loss year 1990 appear to be outlying. This observation has been put aside when estimating parameters. Then the residuals look good.

In figure 3 we can see the estimated time series model for the loss ratio. The red line corresponds to the model expected value. The light blue band indicates estimation uncertainty, one standard deviation up and one standard deviation down. Together with the dark blue band we have the total uncertainty, which is composed of estimation, and stochastic process uncertainty. The points in the graph are the unbiased predictions for the loss ratio by loss year. We can still see prediction intervals for the recent loss years. For the older loss years these have shrunk to nil.

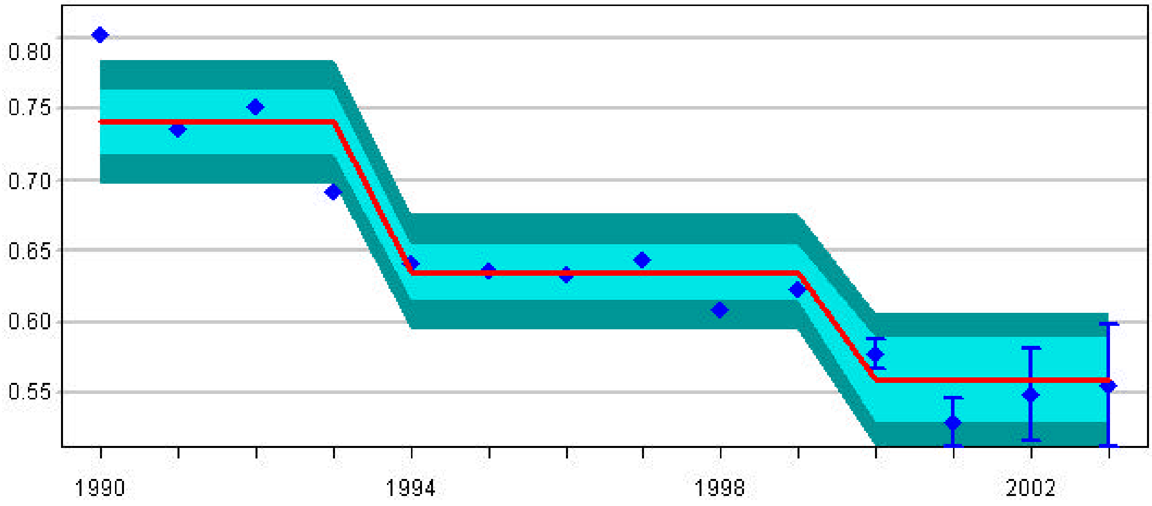

Figure 3: Loss ratio

Figure 4 contains various aspects of the benefit-weighted settlement duration. We can see a curve, points and a histogram. The smooth curve is the power gamma density. The points are empirical counterparts of settlement probabilities in discretised time. The histogram shows the theoretical settlement probabilities together. The effect of estimation uncertainty is shown as roofs on the bars of this histogram.

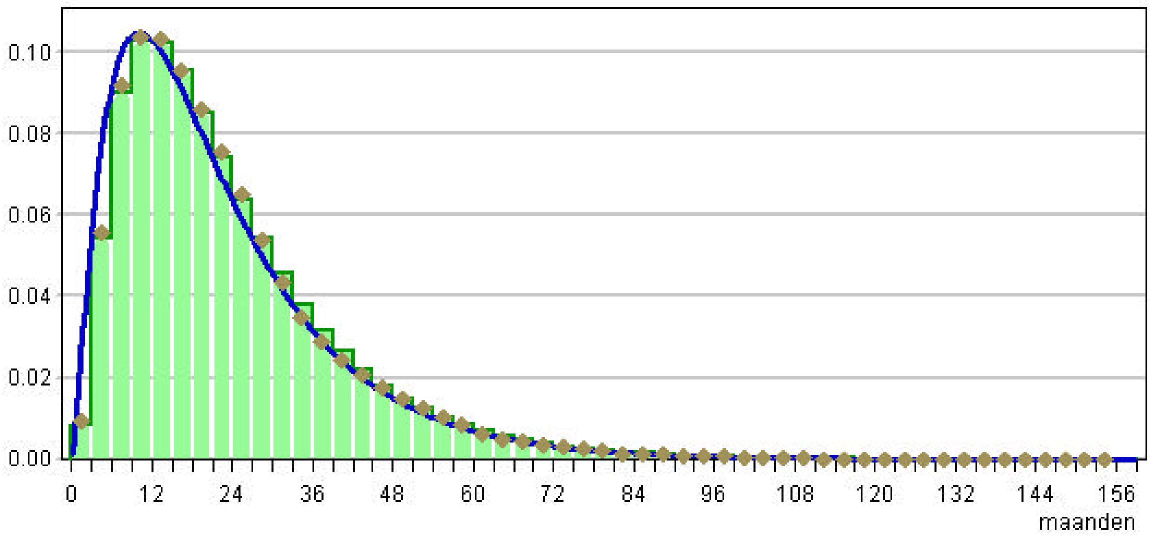

Figure 4: Settlement duration

In the interactive report we can view screens such as cash flow, actuarial loss provision, ultimate loss prediction and completed table. The nominal loss provision has an expected value of 169.9 million and the quantile of order 0.8 gives a prudent provision of 176.3 million.

In the end all truth is channelled through cash flows. Under accounting reconciliation the same model specification has been re-estimated at the beginning of 2002 with the deletion of the observations in the accounting year 2002. In this way we are in a position to simulate profit or loss on the as-if loss provision. At the end of 2001 the model predicted 82.1 million for payments in the accounting year 2002. Quite close, compared with the observed value of 82.5 million. In the case of a prudent provision determined as the quantile of order 0.8, at the end of 2002 we would have seen a profit on the loss years ≤ 2001 of 2.7 million. At the end of 2002 we predict an amount of 48.2 million for payments in 2003 for the loss years ≤ 2002. This then forms the point of departure for the monitoring of the observed payments in 2003, which can also be done quarterly.

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